De MORGAN

FIRST NOTIONS
OF LOGIC

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LOGIC

(PREPARATORY TO THE STUDY OF GEOMETRY)

BY

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The root of all the mischief in the sciences, is this; that falsely magnifying and admiring the powers of the mind, we seek not its real helps.—Bacon.

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*** This Tract contains no more than the author has found, from experience, to be much wanted by students who are commencing with Euclid. It will ultimately form an Appendix to his Treatise on Arithmetic.

The author would not, by any means, in presenting the minimum necessary for a particular purpose, be held to imply that he has given enough of the subject for all the ends of education. He has long regretted the neglect of logic; a science, the study of which would shew many of its opponents that the light esteem in which they hold it arises from those habits of inference which thrive best in its absence. He strongly recommends any student to whom this tract may be the first introduction of the subject, to pursue it to a much greater extent.

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FIRST NOTIONS

OF

LOGIC.

WHAT we here mean by Logic is the examination of that part of reasoning which depends upon the manner in which inferences are formed, and the investigation of general maxims and rules for constructing arguments, so that the conclusion may contain no inaccuracy which was not previously asserted in the premises. It has nothing to do with the truth of the facts, opinions, or presumptions, from which an inference is derived; but simply takes care that the inference shall certainly be true, if the premises be true. Thus, when we say that all men will die, and that all men are rational beings, and thence infer that some rational beings will die, the *logical* truth of this sentence is the same whether it be true or false that men are mortal and rational. This logical truth depends upon the structure of the sentence, and not on the particular matters spoken of. Thus,

Instead of,	Write,
All men will die.	Every A is B.
All men are rational beings.	Every A is C.
Therefore some rational beings will die.	Therefore some Cs are Bs.

The second of these is the same proposition, logically considered, as the first; the consequence in both is virtually contained in, and rightly inferred from, the premises. Whether the premises be true or false, is not a question of logic, but of morals, philosophy, history, or any other knowledge to which their subject-matter belongs: the question of logic is, does the conclusion certainly follow if the premises be true?

Every act of reasoning must mainly consist in comparing together different things, and either finding out, or recalling from previous knowledge, the points in which they resemble or differ from each other. That particular part of reasoning which is called *inference*, consists in the comparison of several and different things with one and the same other thing; and ascertaining the resemblances, or differences, of the several things, by means of the points in which they resemble, or differ from, the thing with which all are compared.

There must then be some propositions already obtained before any inference can be drawn. All propositions are either assertions or denials, and are thus divided into *affirmative* and *negative*. Thus, A is B, and A is not B, are the two forms to which all propositions may be reduced. These are, for our present purpose, the most simple forms; though it will frequently happen that much circumlocution is needed to reduce propositions to them. Thus, suppose the following assertion, 'If he should come to-morrow, he will probably stay till Monday'; how is this to be reduced to the form A is B? There is evidently something spoken of, something said of it, and an affirmative connexion between them. Something, if it happen, that is, the happening of something, makes the happening of another something probable; or *is* one of the things which render the happening of the second thing probable.

A	is	B
The happening of his arrival to-morrow	is	an event from which it may be inferred as probable that he will stay till Monday.

The forms of language will allow the manner of asserting to be varied in a great number of ways; but the reduction to the preceding form is always possible. Thus, 'so he said' is an affirmation, reducible as follows:

What you have just said (or whatever else 'so' refers to)	is	the thing which he said.

By changing 'is' into 'is not,' we make a negative proposition;

but care must always be taken to ascertain whether a proposition which appears negative is really so. The principal danger is that of confounding a proposition which is negative with another which is affirmative of something requiring a negative to describe it. Thus 'he resembles the man who was not in the room,' is affirmative, and must not be confounded with 'he does not resemble the man who was in the room.' Again, 'if he should come to-morrow, it is probable he will not stay till Monday,' does not mean the simple denial of the preceding proposition, but the affirmation of the directly opposite proposition. It is,

A	is	B
The happening of his arrival to-morrow,	is	an event from which it may be inferred to be *im*probable that he will stay till Monday:

whereas the following,

The happening of his arrival to-morrow,	is *not*	an event from which it may be inferred as probable that he will stay till Monday,

would be expressed thus: 'If he should come to-morrow, that is no reason why he should stay till Monday.'

Moreover, the negative words not, no, &c., have two kinds of meaning which must be carefully distinguished. Sometimes they deny, and nothing more: sometimes they are used to affirm the direct contrary. In cases which offer but two alternatives, one of which is necessary, these amount to the same thing, since the denial of one, and the affirmation of the other, are obviously equivalent propositions. In many idioms of conversation, the negative implies affirmation of the contrary in cases which offer not only alternatives, but degrees of alternatives. Thus, to the question, 'Is he tall?' the simple answer, 'No,' most frequently means that he is the contrary of tall, or considerably under the average. But it must be remembered, that, in all logical reasoning, the negation is simply negation, and nothing more, never implying affirmation of the contrary.

The common proposition that two negatives make an affirmative, is true only upon the supposition that there are but two possible things, one of which is denied. Grant that a man must be either able or unable to do a particular thing, and then *not unable* and able are the same things. But if we suppose various degrees of performance, and therefore degrees of ability, it is false, in the common sense of the words, that two negatives make an affirmative. Thus, it would be erroneous to say, 'John is able to translate Virgil, and Thomas is not unable; therefore, what John can do Thomas can do,' for it is evident that the premises mean that John is so near to the best sort of translation that an affirmation of his ability may be made, while Thomas is considerably lower than John, but not so near to absolute deficiency that his ability may be altogether denied. It will generally be found that two negatives imply an affirmative of a weaker degree than the positive affirmation.

Each of the propositions, 'A is B,' and 'A is not B,' may be subdivided into two species: the *universal*, in which every possible case is included; and the *particular*, in which it is not meant to be asserted that the affirmation or negation is universal. The four species of propositions are then as follows, each being marked with the letter by which writers on logic have always distinguished it.

A	*Universal Affirmative*	Every	A is	B
E	*Universal Negative*	No	A is	B
I	*Particular Affirmative*	Some	A is	B
O	*Particular Negative*	Some	A is not	B

In common conversation the affirmation of a part is meant to imply the denial of the remainder. Thus, by 'some of the apples are ripe,' it is always intended to signify that some are not ripe. This is not the case in logical language, but every proposition is intended to make its amount of affirmation or denial, and no more. When we say, 'Some A is B,' or, more grammatically, 'Some As are Bs,' we do not mean to imply that some are not: this may or may not be. Again, the word some means, 'one or more, possibly all.' The following table will shew the bearing of each proposition on the rest.

Every A is B affirms and contains *Some A is B* and denies { *No A is B* / *Some A is not B* }

No A is B affirms and contains *Some A is not B* and denies { *Every A is B* / *Some A is B* }

Some A is B does not contradict { *Every A is B* / *Some A is not B* } but denies *No A is B*

Some A is not B does not contradict { *No A is B* / *Some A is B* } but denies *Every A is B*

Contradictory propositions are those in which one denies *any thing* that the other affirms; *contrary* propositions are those in which one denies *every thing* which the other affirms, or affirms every thing which the other denies. The following pair are contraries.

Every A is B   and   No A is B

and the following are contradictories,

Every A is B	to	Some A is not B
No A is B	to	Some A is B

A contrary, therefore, is a complete and total contradictory; and a little consideration will make it appear that the decisive distinction between contraries and contradictories lies in this, that contraries may both be false, but of contradictories, one must be true and the other false. We may say, ' Either P is true, or *something* in contradiction of it is true;' but we cannot say, ' Either P is true, or *every thing* in contradiction of it is true.' It is a very common mistake to imagine that the *denial* of a proposition gives a right to *affirm* the contrary; whereas it should be, that the *affirmation* of a proposition gives a right to *deny* the contrary. Thus, if we deny that Every A is B, we do not affirm that No A is B, but only that Some A is not B; while, if we affirm that Every A is B, we deny No A is B, and also Some A is not B.

But, as to contradictories, affirmation of one is a denial of the other, and denial of one is affirmation of the other. Thus, either Every A is B, or Some A is not B: affirmation of either is denial of the other, and *vice versâ*.

Let the student now endeavour to satisfy himself of the following. Taking the four preceding propositions, A, E, I, O, let the simple letter signify the affirmation, the same letter in parentheses the denial, and the absence of the letter, that there is neither affirmation nor denial.

From A follow (E), I, (O)		From (A) follow O
From E ...... (A), (I), O		From (E) I
From I ...... (E)		From (I) ...... (A), E, O
From O ...... (A)		From (O) ...... A, (E), I

These may be thus summed up: The affirmation of a universal proposition, and the denial of a particular one, enable us to affirm or deny all the other three; but the denial of a universal proposition, and the affirmation of a particular one, leave us unable to affirm or deny two of the others.

In such propositions as 'Every A is B,' 'Some A is not B,' &c., A is called the *subject*, and B the *predicate*, while the verb 'is' or 'is not,' is called the *copula*. It is obvious that the words of the proposition point out whether the subject is spoken of universally or partially, but not so of the predicate, which it is therefore important to examine. Logical writers generally give the name of *distributed* subjects or predicates to those which are spoken of universally; but as this word is rather technical, I shall say that a subject or predicate enters wholly or partially, according as it is universally or particularly spoken of.

1. In A, or 'Every A is B,' the subject enters wholly, but the predicate only partially. For it obviously says, 'Among the Bs are all the As,' 'Every A is part of the collection of Bs, so that all the As make a part of the Bs, the whole it *may* be.' Thus, 'Every horse is an animal,' does not speak of all animals, but states that all the horses make up a portion of the animals.

2. In E, or 'No A is B,' both subject and predicate enter wholly. 'No A whatsoever is any one out of all the Bs;' 'search the whole collection of Bs, and *every* B shall be found to be something which is not A.'

3. In I, or 'Some A is B,' both subject and predicate enter partially. 'Some of the As are found among the Bs, or make up a part (the whole possibly, but not known from the preceding) of the Bs.'

4. In O, or 'Some A is not B,' the subject enters partially, and the predicate wholly. 'Some As are none of them any whatsoever of the Bs; every B will be found to be no one out of a certain portion of the As.'

It appears then that,

In affirmatives, the predicate enters partially.

In negatives, the predicate enters wholly.

In contradictory propositions, both subject and predicate enter differently in the two.

The *converse* of a proposition is that which is made by interchanging the subject and predicate, as follows :

	The proposition.	Its converse.
A	Every A is B	Every B is A
E	No A is B	No B is A
I	Some A is B	Some B is A
O	Some A is not B	Some B is not A

Now, it is a fundamental and self-evident proposition, that no consequence must be allowed to assert more widely than its premises; so that, for instance, an assertion which is only of some Bs can never lead to a result which is true of all Bs. But if a proposition assert agreement or disagreement, any other proposition which asserts the same, to the same extent and no further, must be a legitimate consequence; or, if you please, must amount to the whole, or part, of the original assertion in another form. Thus, the converse of A is not true: for, in 'Every A is B,' the predicate enters partially; while in 'Every B is A,' the subject enters wholly. 'All the As make up a part of the Bs, then a part of the Bs are among the As, or some B is A.' Hence, the only *legitimate* converse of 'Every A is B' is, 'Some B is A.' But in 'No A is B,' both subject and predicate enter wholly, and 'No B is A' is, in fact, the same proposition as 'No A is B.' And 'Some A is B' is also the same as its converse 'Some B is A:' here both terms enter partially. But 'Some A is not B' admits of no converse whatever; it is perfectly consistent with all assertions upon B and A in which B is the subject. Thus neither of the four following lines is inconsistent with itself.

Some A is not B and Every B is A
Some A is not B and No B is A
Some A is not B and Some B is A
Some A is not B and Some B is not A.

We find then, including converses, which are not identical with their direct propositions, *six* different ways of asserting or denying, with respect to agreement or non-agreement, total or partial, between A and, say X: these we write down, designating the additional assertions by U and Y.

	Identical.	Identical.	
A Every A is X	E No A is X	I Some A is X	O Some A is not X
U Every X is A	No X is A	Some X is A	Y Some X is not A

We shall now repeat and extend the table of page 8 (A), &c., meaning, as before, the denial of A, &c.

From A or (O)	follow	A,	(E),	I	(O)		
From E or (I)	......	(A),	E,	(I),	O,	(U),	Y
From I or (E)	......		(E)	I			
From O or (A)	......	(A),			O		
From U or (Y)	......		(E)	I,		U	(Y)
From Y or (U)	......					(U)	Y

Having thus discussed the principal points connected with the simple assertion, we pass to the manner of making two assertions give a third. Every instance of this is called a syllogism, the two assertions which form the basis of the third are called premises, and the third itself the conclusion.

If two things both agree with a third in any particular, they agree with each other in the same; as, if A be of the same colour as X, and B of the same colour as X, then A is of the same colour as B. Again, if A differ from X in any particular in which B agrees with X, then A and B differ in that particular. If A be not of the same colour as X, and B be of the same colour as X, then A is not of the colour of B. But if A and B both differ from X in any particular, nothing can be inferred; they may either differ in the same way and to the same extent, or not. Thus, if A and B be both of different colours from X, it neither follows that they agree, nor differ, in their own colours.

The paragraph preceding contains the essential parts of all inference, which consists in comparing two things with a third, and finding from their agreement or difference with that third, their agreement or difference with one another. Thus, Every A is X, every B is X, allows us to infer that A and B have all those qualities in common which are necessary to X. Again, from Every A is X, and 'No B is X,' we infer that A and B differ from one another in all particulars which are essential to X. The preceding forms, however, though they represent common reasoning better than the ordinary syllogism, to which we are now coming, do not constitute the ultimate forms of inference. Simple *identity* or *non-identity* is the ultimate state to which every assertion may be reduced; and we shall, therefore, first ask, from what identities, &c., can other identities, &c., be produced? Again, since we name objects in species, each species consisting of a number of individuals, and since our assertion may include all or only part of a species, it is further necessary to ask, in every instance, to what extent the conclusion drawn is true, whether of all, or only of part?

Let us take the simple assertion, 'Every living man respires;' or, every living man is one of the things (however varied they may be) which respire. If we were to inclose all living men in a large triangle, and all respiring objects in a large circle, the preceding assertion, if true, would require that the whole of the triangle should be contained in the circle. And in the same way we may reduce any assertion to the expression of a coincidence, total or partial, between two figures. Thus, a point in a circle may represent an individual of one species, and a point in a triangle an individual of another species: and we may express that the whole of one species is asserted to be contained or not contained in the other by such forms as, 'All the △ is in the ○'; 'None of the △ is in the ○'.

Any two assertions about A and B, each expressing agreement or disagreement, total or partial, with or from X, and leading to a conclusion with respect to A or B, is called a syllogism, of which X is called the *middle term*. The plainest syllogism is the following:—

	Every A is X		All the △ is in the ○
	Every X is B		All the ○ is in the □
Therefore	Every A is B	Therefore	All the △ is in the □

In order to find all the possible forms of syllogism, we must make a table of all the elements of which they can consist; namely—

A and X		B and X
Every A is X	A	Every B is X
No A is X	E	No B is X
Some A is X	I	Some B is X
Some A is not X	O	Some B is not X
Every X is A	U	Every X is B
Some X is not A	Y	Some X is not B

Or their synonymes,

△ and ○		□ and ○
All the △ is in the ○	A	All the □ is in the ○
None of the △ is in the ○	E	None of the □ is in the ○
Some of the △ is in the ○	I	Some of the □ is in the ○
Some of the △ is not in the ○	O	Some of the □ is not in the ○
All the ○ is in the △	U	All the ○ is in the □
Some of the ○ is not in the △	Y	Some of the ○ is not in the □

Now, taking any one of the six relations between A and X, and combining it with either of those between B and X, we have six pairs of premises, and the same number repeated for every different relation of A and X. We have then thirty-six forms to consider: but, thirty of these (namely, all but (A, A) (E, E), &c.) are half of them repetitions of the other half. Thus, ' Every A is X, no B is X,' and ' Every B is X, no A is X,' are of the same form, and only differ by changing A into B and B into A. There are then only 15 + 6, or 21 distinct forms, some of which give a necessary conclusion, while others do not. We shall select the former of these, classifying them by their conclusions; that is, according as the inference is of the form A, E, I, or O.

I. In what manner can a universal affirmative conclusion be drawn; namely, that one figure is entirely contained in the other? This we can only assert when we know that one figure is entirely contained in the circle, which itself is entirely contained in the other figure. Thus,

Every A is X	All the △ is in the ○	A
Every X is B	All the ○ is in the □	A
∴ Every A is B	∴ All the △ is in the □	A

is the only way in which a universal affirmative conclusion can be drawn.

II. In what manner can a universal negative conclusion be drawn; namely, that one figure is entirely exterior to the other? Only when we are able to assert that one figure is entirely within, and the other entirely without, the circle. Thus,

Every A is X	All the △ is in the ○	A
No B is X	None of the □ is in the ○	E
∴ No A is B	None of the △ is in the □	E

is the only way in which a universal negative conclusion can be drawn.

III. In what manner can a particular affirmative conclusion be drawn; namely, that part or all of one figure is contained in the other? Only when we are able to assert that the whole circle is part of one of the figures, and that the whole, or part of the circle, is part of the other figure. We have then two forms.

Every X is A	All the ○ is in the △	A
Every X is B	All the ○ is in the □	A
∴ Some A is B	∴ Some of the △ is in the □	I

Every X is A	All the ○ is in the △	A
Some X is B	Some of the ○ is in the □	I
Some A is B	Some of the △ is in the □	I

The second of these contains all that is strictly necessary to the conclusion, and the first may be omitted. That which follows when an assertion can be made as to some, must follow when the same assertion can be made of all.

IV. How can a particular negative proposition be inferred; namely, that part, or all of one figure, is not contained in the other? It would seem at first sight, whenever we are able to assert that part or all of one figure is in the circle, and that part or all of the other figure is not. The weakest syllogism from which such an inference can be drawn would then seem to be as follows.

Some A is X	Some of the △ is in the ○
Some B is not X	Some of the □ is not in the ○
∴ Some B is not A	∴ Some of the △ is not in the □

But here it will appear, on a little consideration, that the conclusion is only thus far true; that those As which are Xs cannot be *those* Bs which are not Xs; but they may be *other* Bs, about which nothing is asserted when we say that *some* Bs are not Xs. And further consideration will make it evident, that a conclusion of this form can only be arrived at when one of the figures is entirely within the circle, and the whole or part of the other without; or else when the whole of one of the figures is without the circle, and the whole or part of the other within; or lastly, when the circle lies entirely within one of the figures, and not entirely within the other. That is, the following are the distinct forms which allow of a particular negative conclusion, in which it should be remembered that a particular proposition in the premises may always be changed into a universal one, without affecting the conclusion. For that which necessarily follows from "some," follows from "all."

Every A is X	All the △ is in the ○	A
Some B is not X	Some of the □ is not in the ○	O
∴ Some B is not A	Some of the □ is not in the △	O

No A is X	None of the △ is in the ○	E
Some B is X	Some of the □ is in the ○	I
∴ Some B is not A	Some of the □ is not in the △	O
Every X is A	All the ○ is in the △	A
Some X is not B	Some of the ○ is not in the □	O
∴ Some A is not B	Some of the △ is not in the □	O

It appears, then, that there are but six distinct syllogisms. All others are made from them by strengthening one of the premises, or converting one or both of the premises, where such conversion is allowable; or else by first making the conversion, and then strengthening one of the premises. And the following arrangement will shew that two of them are universal, three of the others being derived from them by weakening one of the premises in a manner which does not destroy, but only weakens, the conclusion.

1. Every A is X
   Every X is B
   ———
   Every A is B

3. Every A is X
   No B is X
   ———
   No A is B

....

2. Some A is X
   Every X is B
   ———
   Some A is B

4. Some A is X
   No B is X
   ———
   Some A is not B

5. Every A is X
   Some B is not X
   ———
   Some B is not A

6. Every X is A
   Some X is not B
   ———
   Some A is not B

We may see how it arises that one of the partial syllogisms is not immediately derived, like the others, from a universal one. In the preceding, A E E may be considered as derived from A A A, by changing the term in which X enters universally into its contrary. If this be done with the other term instead, we have

No A is X
Every X is B } from which universal premises we cannot deduce a universal conclusion, but only Some B is not A.

If we weaken one and the other of these premises, as they stand, we obtain

Some A is not X		No A is X
Every X is B	and	Some X is B
No conclusion		Some B is not A

equivalent to the fourth of the preceding: but if we convert the first premiss, and proceed in the same manner,

	No X is A		Some X is not A
From	Every X is B	we obtain	Every X is B
	Some B is not A		Some B is not A

which is legitimate, and is the same as the last of the preceding list, with A and B interchanged.

Before proceeding to shew that all the usual forms are contained in the preceding, let the reader remark the following rules, which may be proved either by collecting them from the preceding cases, or by independent reasoning.

1. The middle term must enter universally into one or the other premiss. If it were not so, the one premiss might speak of one part of the middle term, and the other of the other; so that there would, in fact, be no middle term. Thus, 'Every A is X, Every B is X,' gives no conclusion: it may be thus stated;

All the As make up *a part* of the Xs
All the Bs make up *a part* of the Xs

And, before we can know that there is any common term of comparison at all, we must have some means of shewing that the two parts are the same; or the preceding premises by themselves are inconclusive.

2. No term must enter the conclusion more generally than it is found in the premises; thus, if A be spoken of partially in the premises, it must enter partially into the conclusion. This is obvious, since the conclusion must assert no more than the premises imply.

3. From premises both negative no conclusion can be drawn. For it is obvious, that the mere assertion of disagreement between each of two things and a third, can be no reason for inferring either agreement or disagreement between these two things. It will not be difficult to reduce any case which falls under this rule to a breach of the first rule: thus, No A is X, No B is X, gives

Every A is (something which is not X)
Every B is (something which is not X)

in which the middle term is not spoken of universally in either. Again, 'No X is A, Some X is not B,' may be converted into

Every A is (a thing which is not X)
Some (thing which is not B) is X

in which there is no middle term.

4. From premises both particular no conclusion can be drawn. This is sufficiently obvious when the first or second rule is broken, as in 'Some A is X, Some B is X.' But it is not immediately obvious when the middle term enters one of the premises universally. The following reasoning will serve for exercise in the preceding results. Since both premises are particular in form, the middle term can only enter one of them universally by being the predicate of a negative proposition; consequently (Rule 3) the other premiss must be affirmative, and, being particular, neither of its terms is universal. Consequently both the terms as to which the conclusion is to be drawn enter partially, and the conclusion (Rule 2) can only be a particular *affirmative* proposition. But if one of the premises be negative, the conclusion must be *negative* (as we shall immediately see). This contradiction shews that the supposition of particular premises producing a legitimate result is inadmissible.

5. If one premiss be negative, the conclusion, if any, must be negative. If one term agree with a second and disagree with a third, no agreement can be inferred between the second and third.

6. If one premiss be particular, the conclusion must be particular. This is not very obvious, since the middle term may be universally spoken of in a particular proposition, as in Some B is not X. But this requires one negative proposition, whence (Rule 3) the other must be affirmative. Again, since the conclusion must be negative (Rule 5) its predicate is spoken of universally, and, therefore, must enter universally; the other term A must enter, then, in a universal affirmative proposition, which is against the supposition.

In the preceding set of syllogisms we observe one form only which produces A, or E, or I, but three which produce O.

Let an assertion be said to be weakened when it is reduced from universal to particular, and strengthened in the contrary case. Thus, 'Every A is B' is called stronger than 'Some A is B.'

Every form of syllogism which can give a legitimate result is either one of the preceding six, or another formed from one of the six, either by changing one of the assertions into its converse, if that be allowable, or by strengthening one of the premises without altering the conclusion, or both. Thus,

Some A is X Every X is B	may be written	Some X is A Every X is B
	What follows will still follow from	*Every* X is A Every X is B

for all which is true when 'Some X is A,' is not less true when 'Every X is A.'

It would be possible also to form a legitimate syllogism by weakening the conclusion, when it is universal, since that which is true of all is true of some. Thus, 'Every A is X, Every X is B,' which yields 'Every A is B,' also yields 'Some A is B.' But writers on logic have always considered these syllogisms as useless, conceiving it better to draw from any premises their strongest conclusion. In this they were undoubtedly right; and the only question is, whether it would not have been advisable to make the premises as weak as possible, and not to admit any syllogisms in which more appeared than was absolutely necessary to the conclusion. If such had been the practice, then

Every X is A, Every X is B, therefore Some A is B

would have been considered as formed by a spurious and unnecessary excess of assertion. The minimum of assertion would be contained in either of the following,

Every X is A, Some X is B, therefore Some A is B
Some X is A, Every X is B, therefore Some A is B

In this tract, syllogisms have been divided into two classes: first, those which prove a universal conclusion; secondly, those which prove a partial conclusion, and which are (all but one) derived from the first by weakening one of the premises, in such manner as to produce a legitimate but weakened conclusion. Those of the first class are placed in the first column, and the other in the second.

	Universal.		Particular.	
A	Every A is X		Some A is X	I
A	Every X is B	——	Every X is B	A
A	Every A is B		Some A is B	I
			Some A is X	I
			No X is B	E
A	Every A is X		Some A is not B	O
E	No X is B	——	Every A is X	A
E	No A is B		Some B is not X	O
			Some B is not A	O
			Every X is A	A
	....		Some X is not B	O
			Some A is not B	O

In all works on logic, it is customary to write that premiss first which contains the predicate of the conclusion. Thus,

Every X is B		Every A is X
Every A is X	would be written, and not	Every X is B
Every A is B		Every A is B

The premises thus arranged are called major and minor; the predicate of the conclusion being called the major term, and its subject the minor. Again, in the preceding case we see the various subjects coming in the order X, B; A, X; A, B: and the number of different orders which can appear is four, namely —

X B	B X	X B	B X
A X	A X	X A	X A
A B	A B	A B	A B

which are called the four figures, and every kind of syllogism in each figure is called a mood. I now put down the various moods of each figure, the letters of which will be a guide to find out those of the preceding list from which they are derived. Co means that a premiss of the preceding list has been converted; + that it has been strengthened; Co +, that both changes have taken place. Thus,

A	Every X is B		A	Every X is B
I	Some A is X	becomes	A	Every X is A : (Co +)
I	Some A is B		I	Some A is B

And Co + abbreviates the following: If some A be X, then some X is A (Co); and all that is true when Some X is A, is true when Every X is A (+); therefore the second is legitimate, if the first be so.

First Figure.

A	Every X is B	A	Every X is B
A	Every A is X	I	Some A is X
A	Every A is B	I	Some A is B
E	No X is B	E	No X is B
A	Every A is X	I	Some A is X
E	No A is B	O	Some A is not B

Second Figure.

E	No B is X (Co)	E	No B is X (Co)
A	Every A is X	I	Some A is X
E	No A is B	O	Some A is not B
A	Every B is X	A	Every B is X
E	No A is X (Co)	O	Some A is not X
E	No A is B	O	Some A is not B

Third Figure.

A	Every X is B	E	No X is B
A	Every X is A (Co +)	A	Every X is A (Co +)
I	Some A is B	O	Some A is not B
I	Some X is B (Co)	O	Some X is not B
A	Every X is A	A	Every X is A
I	Some A is B	O	Some A is not B
A	Every X is B	E	No X is B
I	Some X is A (Co)	I	Some X is A (Co)
I	Some A is B	O	Some A is not B

Fourth Figure.

A	Every B is X (+)	I	Some B is X
A	Every X is A	A	Every X is A
I	Some A is B	I	Some B is A
A	Every B is X	E	No B is X (Co)
E	No X is A	A	Every X is A (Co +)
E	No A is B	O	Some A is not B

E	No B is X (Co)
I	Some X is A (Co)
O	Some A is not B

The above is the ancient method of dividing syllogisms; but, for the present purpose, it will be sufficient to consider the six from which the rest can be obtained. And since some of the six have A in the predicate of the conclusion, and not B, we shall join to them the six other syllogisms which are found by transposing B and A. The complete list, therefore, of syllogisms with the weakest premises and the strongest conclusions, in which a comparison of A and B is obtained by comparison of both with X, is as follows:

Every A is X	Every B is X	Some A is X	Some B is X
Every X is B	Every X is A	No X is B	No X is A
Every A is B	Every B is A	Some A is not B	Some B is not A
Every A is X	Every B is X	Every A is X	Every B is X
No X is B	No X is A	Some B is not X	Some A is not X
No A is B	No B is A	Some B is not A	Some A is not B
Some A is X	Some B is X	Every X is A	Every X is B
Every X is B	Every X is A	Some X is not B	Some X is not A
Some A is B	Some B is A	Some A is not B	Some B is not A

In the list of page 19, there was nothing but recapitulation of forms, each form admitting a variation by interchanging A and B. This interchange having been made, and the results collected as above, if we take every case in which B is the predicate, or can be made the predicate by allowable conversion, we have a collection of all possible *weakest* forms in which the result is one of the four 'Every A is B,' 'No A is B,' 'Some A is B,' 'Some A is not B'; as follows. The premises are written in what appeared the most natural order, without distinction of major or minor; and the letters prefixed are according to the forms of the several premises, as in page 10.

A	Every A is X
U	Every X is B
A	Every A is B

I	Some A is X	I	Some B is X
U	Every X is B	U	Every X is A
I	Some A is B	I	Some A is B
A	Every A is X	A	Every B is X
E	No B is X	E	No A is X
E	No A is B	E	No A is B

I	Some A is X	A	Every B is X	U	Every X is A
E	No B is X	O	Some A is not X	Y	Some X is not B
O	Some A is not B	O	Some A is not B	O	Some A is not B

Every assertion which can be made upon two things by comparison with any third, that is, every simple inference, can be reduced to one of the preceding forms. Generally speaking, one of the premises is omitted, as obvious from the conclusion; that is, one premiss being named and the conclusion, that premiss is implied which is necessary to make the conclusion good. Thus, if I say, "That race must have possessed some of the arts of life, for they came from Asia," it is obviously meant to be asserted, that all races coming from Asia must have possessed some of the arts of life. The preceding is then a syllogism, as follows:

'That race' is 'a race of Asiatic origin:'
Every 'race of Asiatic origin' is 'a race which must have possessed some of the arts of life:'
Therefore, That race *is* a race which must have possessed some of the arts of life.

A person who makes the preceding assertion either means to imply, antecedently to the conclusion, that all Asiatic races must have possessed arts, or he talks nonsense if he asserts the conclusion positively. 'A must be B, for it is X,' can only be true when 'Every X is B.' This latter proposition may be called the suppressed premiss; and it is in such suppressed propositions that the greatest danger of error lies. It is also in such propositions that men convey opinions which they would not willingly express. Thus, the honest witness who said, 'I always thought him a respectable man—he kept his gig,' would probably not have admitted in direct terms, 'Every man who keeps a gig must be respectable.'

I shall now give a few detached illustrations of what precedes.

"His imbecility of character might have been inferred from his proneness to favourites; for all weak princes have this failing." The preceding would stand very well in a history, and many would pass it over as containing very good inference. Written, however, in the form of a syllogism, it is,

All weak princes are prone to favourites
He was prone to favourites

Therefore He was a weak prince

which is palpably wrong. (Rule 1.) The writer of such a sentence as the preceding might have meant to say, 'for all who have this failing are weak princes;' in which case he would have inferred rightly. Every one should be aware that there is much false inference arising out of badness of style, which is just as injurious to the habits of the untrained reader as if the errors were mistakes of logic in the mind of the writer.

'A is less than B; B is less than C: therefore A is less than C.' This, at first sight, appears to be a syllogism; but, on reducing it to the usual form, we find it to be,

A is (a magnitude less than B)
B is (a magnitude less than C)
Therefore A is (a magnitude less than C)

which is not a syllogism, since there is no middle term. Evident as the preceding is, the following additional proposition must be formed before it can be made explicitly logical. 'If B be a magnitude less than C, then every magnitude less than B is also less than C.' There is, then, before the preceding can be reduced to a syllogistic form, the necessity of a deduction from the second premiss, and the substitution of the result instead of that premiss. Thus,

A is less than B
Less than B is less than C: following from B is less than C.

Therefore A is less than C

But, if the additional argument be examined—namely, if B be less than C, then that which is less than B is less than C—it will be found to require precisely the same considerations repeated; for the original inference was nothing more. In fact, it may easily be seen as follows, that the proposition before us involves more than any simple syllogism can express. When we say that A is less than B, we say that if A were applied to B, every part of A would match a part of B, and there would be parts of B remaining over. But when we say, 'Every A is B,' meaning the premiss of a common syllogism, we say that every instance of A is an instance of B, without saying any thing as to whether there are or are not instances of B still left, after those which are also A are taken away. If, then, we wish to write an ordinary syllogism in a manner which shall correspond with 'A is less than B, B is less than C, therefore A is less than C,' we must introduce a more definite amount of assertion than was made in the preceding forms. Thus,

	Every A is B, and there are Bs which are not As
	Every B is C, and there are Cs which are not Bs
Therefore	Every A is C, and there are Cs which are not As

Or thus:

The Bs contain all the As, and more
The Cs contain all the Bs, and more

The Cs contain all the As, and more

The most technical form, however, is,

From	Every A is B;	[Some B is not A]
	Every B is C;	[Some C is not B]
Follows	Every A is C;	[Some C is not A]

This sort of argument is called *à fortiori* argument, because the premises are more than sufficient to prove the conclusion, and the extent of the conclusion is thereby greater than its mere form would indicate. Thus, 'A is less than B, B is less than C, therefore, *à fortiori*, A is less than C,' means that the extent to which A is less than C must be greater than that to which A is less than B, or B than C. In the syllogism last written, either of the bracketted premises might be struck out without destroying the conclusion; which last would, however, be weakened. As it stands, then, the part of the conclusion, 'Some C is not A,' follows *à fortiori*.

The argument *à fortiori*, may then be defined as a universally affirmative syllogism, in which both of the premises are shewn to be less than the whole truth, or greater. Thus, in 'Every A is X, Every X is B, therefore Every A is B,' we do not certainly imply that there are more Xs than As, or more Bs than Xs, so that we do not know that there are more Bs than As. But if we are at liberty to state the syllogism as follows,

All the As make up part (and part only) of the Xs
Every X is B;

then we are certain that

All the As make up part (and part only) of the Bs.

But if we are at liberty further to say that

All the As make up part (and part only) of the Xs
All the Xs make up part (and part only) of the Bs

then we conclude that

All the As make up *part of part* (only) of the Bs

and the words in Italics mark that quality of the conclusion from which the argument is called *à fortiori.*

Most syllogisms which give an affirmative conclusion are generally meant to imply *à fortiori* arguments, except only in mathematics. It is seldom, except in the exact sciences, that we meet with a proposition, 'Every A is B,' which we cannot immediately couple with 'Some Bs are not As.'

When an argument is completely established, with the exception of one assertion only, so that the inference may be drawn as soon as that one assertion is established, the result is stated in a form which bears the name of an *hypothetical* syllogism. The word hypothesis means nothing but supposition; and the species of syllogism just mentioned first lays down the assertion that a consequence will be true if a certain condition be fulfilled, and then either asserts the fulfilment of the condition, and thence the consequence, or else denies the consequence, and thence denies the fulfilment of the condition. Thus, if we know that

When A is B, it follows that P is Q;

then, as soon as we can ascertain that A is B, we can conclude that P is Q; or, if we can shew that P is not Q, we know that A is not B. But if we find that A is not B, we can infer nothing; for the preceding does not assert that P is Q *only* when A is B. And if we find out that P is Q, we can infer nothing. This conditional syllogism may be converted into an ordinary syllogism, as follows. Let K be any 'case in which A is B,' and Z a 'case in which P is Q'; then the preceding assertion amounts to 'Every K is Z.' Let L be a particular instance, the A of which may or may not be B. If A be B in the instance under discussion, or if A be not B, we have, in the one case and the other,

	Every K is Z	Every K is Z
	L is a K	L is not a K
Therefore	L is a Z	No conclusion

Similarly, according as a particular case (M) is or is not Z, we have

Every K is Z	Every K is Z
M is a Z	M is not a Z
No conclusion	M is not a K

That is to say: The assertion of an hypothesis is the assertion of its necessary consequence, and the denial of the necessary consequence is the denial of the hypothesis; but the assertion of the necessary consequence gives no right to assert the hypothesis, nor does the denial of the hypothesis give any right to deny the truth of that which would (were the hypothesis true) be its necessary consequence.

Demonstration is of two kinds: which arises from this, that every proposition has a contradictory; and of these two, one must be true and the other must be false. We may then either prove a proposition to be true, or its contradictory to be false. 'It is true that Every A is B,' and, 'it is false that there are some As which are not Bs,' are the same proposition; and the proof of either is called the indirect proof of the other.

But how is any proposition to be proved false, except by proving a contradiction to be true? By proving a necessary consequence of the proposition to be false. But this is not a complete answer, since it involves the necessity of doing the same thing; or, so far as this answer goes, one proposition cannot be proved false unless by proving another to be false. But it may happen, that a necessary consequence can be obtained which is obviously and self-evidently false, in which case no further proof of the falsehood of the hypothesis is necessary. Thus the proof which Euclid gives that all equiangular triangles are equilateral is of the following structure, logically considered.

(1.) If there be an equiangular triangle not equilateral, it follows that a whole can be found which is not greater than its part.*

(2.) It is false that there can be any whole which is not greater than its part (self evident).

(3.) Therefore it is false that there is any equiangular triangle which is not equilateral; or all equiangular triangles are equilateral.

When a proposition is established by proving the truth of the matters it contains, the demonstration is called *direct;* when by proving the falsehood of every contradictory proposition, it is called *indirect.* The latter species of demonstration is as logical as the former, but not of so simple a kind; whence it is desirable to use the former whenever it can be obtained.

The use of indirect demonstration in the Elements of Euclid is almost entirely confined to those propositions in which the converses of simple propositions are proved. It frequently happens that an established assertion of the form

Every A is B .... (1)

may be easily made the means of deducing,

Every (thing not A) is not B ... (2)

which last gives

Every B is A .... (3)

* This is the proposition in proof of which nearly the whole of the demonstration of Euclid is spent.

The conversion of the second proposition into the third is usually made by an indirect demonstration, in the following manner. If possible, let there be one B which is not A, (2) being true. Then there is one thing which is not A and is B; but every thing not A is not B; therefore there is one thing which is B and is not B: which is absurd. It is then absurd that there should be one single B which is not A; or, Every B is A.

The following proposition contains a method which is of frequent use.

Hypothesis.—Let there be any number of propositions or assertions,—three for instance, A, B, and C,—of which it is the property that one or the other must be true, *and one only*. Let there be three other propositions, P, Q, and R, of which it is also the property that one, and one only, must be true. Let it also be a connexion of those assertions, that

When A is true, P is true
When B is true, Q is true
When C is true, R is true

Consequence: then it follows that

When P is true, A is true
When Q is true, B is true
When R is true, C is true

For, when P is true, then Q and R must be false; consequently, neither B nor C can be true, for then Q or R would be true. But either A, B, or C must be true, therefore A must be true; or, when P is true, A is true. In a similar way the remaining assertions may be proved.

Case 1. If	When P is Q,	A is B
	When P is not Q,	A is not B
It follows that	When A is B,	P is Q
	When A is not B,	P is not Q

Case 2. If
- When A is greater than B, P is greater than Q
- When A is equal to B, P is equal to Q
- When A is less than B, P is less than Q

It follows that	When P is greater than Q, A is greater than B	
	When P is equal to Q, A is equal to B	
	When P is less than Q, A is less than B	

We have hitherto supposed that the premises are actually true; and, in such a case, the logical conclusion is as certain as the premises. It remains to say a few words upon the case in which the premises are probably, but not certainly, true.

The probability of an event being about to happen, and that of an argument being true, may be so connected that the usual method of measuring the first may be made to give an easy method of expressing the second. Suppose an urn, or lottery, with a large number of balls, black or white; then, if there be twelve white balls to one black, we say it is twelve to one that a white ball will be drawn, or that a white ball is twelve times as probable as a black one. A certain assertion may be in the same condition as to the force of probability with which it strikes the mind: that is, the questions

Is the assertion true?
Will a white ball be drawn?

may be such that the answer, 'most probably,' expresses the same degree of likelihood in both cases.

We have before explained that logic has nothing to do with the truth or falsehood of assertions, but only professes, supposing them true, to collect and classify the legitimate methods of drawing inferences. Similarly, in this part of the subject, we do not trouble ourselves with the question, How are we to find the probability due to premises? but we ask: Supposing (happen how it may) that we *have* found the probability of the premises, required the probability of the conclusion. When the odds in favour of a conclusion are, say 6 to 1, there are, out of every 7 possible chances, 6 in favour of the conclusion, and 1 against it. Hence $\frac{6}{7}$ and $\frac{1}{7}$ will represent the proportions, for and against, of all the possible cases which exist.

Thus we have the succession of such results as in the following table:—

Odds in favour of an event	Probability for	Probability against
1 to 1	$\frac{1}{2}$	$\frac{1}{2}$
2 to 1	$\frac{2}{3}$	$\frac{1}{3}$
3 to 1	$\frac{3}{4}$	$\frac{1}{4}$
3 to 2	$\frac{3}{5}$	$\frac{2}{5}$
4 to 1	$\frac{4}{5}$	$\frac{1}{5}$
4 to 3	$\frac{4}{7}$	$\frac{3}{7}$
5 to 1	$\frac{5}{6}$	$\frac{1}{6}$
&c.	&c.	&c.

Let the probability of a conclusion, as derived from the premises (that is on the supposition that it was never imagined to be possible till the argument was heard), be called the *intrinsic probability* of the argument. This is found by multiplying together the probabilities of all the assertions which are necessary to the argument. Thus, suppose that a conclusion was held to be impossible until an argument of a single syllogism was produced, the premises of which have severally five to one and eight to one in their favour. Then $\frac{5}{6} \times \frac{8}{9}$, or $\frac{40}{54}$, is the intrinsic probability of the argument, and the odds in its favour are 40 to 14, or 20 to 7.

But this intrinsic probability is not always that of the conclusion; the latter, of course, depending in some degree on the likelihood which the conclusion was supposed to have before the argument was produced. A syllogism of 20 to 7 in its favour, advanced in favour of a conclusion which was beforehand as likely as not, produces a much more probable result than if the conclusion had been thought absolutely false until the argument produced a certain belief in the possibility of its being true.

The change made in the probability of a conclusion by the introduction of an argument (or of a new argument, if some have already preceded) is found by the following rule.

From the sum of the existing probability of the conclusion and the intrinsic probability of the new argument, take their product; the remainder is the probability of the conclusion, as reinforced by the argument. Thus, $a + b - ab$ is the probability of the truth of a conclusion after the introduction of an argument of the intrinsic probability b, the previous probability of the said conclusion having been a.

Thus, a conclusion which has at present the chance $\frac{2}{3}$ in its favour, when reinforced by an argument whose intrinsic probability is $\frac{3}{4}$, acquires the probability $\frac{2}{3} + \frac{3}{4} - \frac{2}{3} \times \frac{3}{4}$ or, $\frac{2}{3} + \frac{3}{4} - \frac{1}{2}$, or $\frac{11}{12}$; or, having 2 to 1 in its favour before, it has 11 to 1 in its favour after, the argument.

When the conclusion was neither likely nor unlikely beforehand (or had the probability $\frac{1}{2}$), the shortest way of applying the preceding rule (in which $a + b - ab$ becomes $\frac{1}{2} + \frac{1}{2} b$) is to divide the sum of the numerator and denominator of the intrinsic probability of the argument by twice the denominator. Thus, an argument of which the intrinsic probability is $\frac{3}{4}$, gives to a conclusion on which no bias previously existed, the probability $\frac{7}{8}$ or $\frac{3+4}{2 \times 4}$.

THE END.

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